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A Method for Comparing Ballistic and Electric Propulsion Performance

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Nomenclature

B	=	area of ballistic superiority
b,e,f	=	cost coefficients
C	=	boundary of equal cost
H	=	area of hybrid superiority
M_B	=	Earth orbital mass of ballistic system
M_H	=	Earth orbital mass of hybrid system
M_W	=	mass of propulsive powerplant
n	=	scaling exponent
P	=	power level
τ	=	minimum energy flight time

Introduction

THREE primary benefits are attributed to the use of nuclear-electric upper stages: reduction of trip time to the outer planets, increase in payload capacity beyond low Earth orbit, or reduction of launch vehicle requirements for a given payload.[1] To what extent one or more of these potential benefits may be realized depends on the applied groundrules, on powerplant specific mass, and on cost considerations.[2]

This Note describes a method which permits systematic exploration of the effects of the groundrules, the uncertainties in the performance parameters, and the major cost tradeoff factors which will later influence economic comparisons between space propulsion systems exhibiting widely differing characteristics. The two different systems singled out for treatment in this paper are the "ballistic" (consisting of high-thrust stages only) and the "hybrid" (consisting of combination of high-thrust and nuclear-electric stages).

Method

The general approach begins with specification of the target and the gross magnitude of the payload. Mission groundrules include capture orbit characteristics and allowable trip time. The trajectory problem and the systems optimization problem are solved in iterative fashion for both the hybrid and the ballistic case. For the examples presented in this particular Note, this has been accomplished by the use of a very fast numerical code.[3] The points at which the hybrid and the ballistic systems produce the same results constitute boundaries of equal performance. Some consideration will also be given to certain cost tradeoff functions.

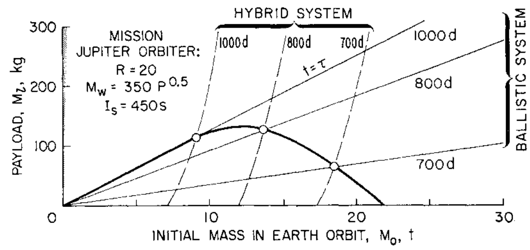

Fig. 1 Boundary of equal performance between hybrid and ballistic systems.

For planetary missions, the characteristics of the capture orbit strongly influence the propulsive requirements; and since the trip time/payload characteristics of high-thrust and low-thrust systems are radically different, the capture orbit and trip time groundrules can strongly influence the outcome of the comparison. Ballistic systems generally exhibit a trend in which the inert mass fraction decreases with increasing mass of the stage but increases with increasing specific impulse. For the sake of simplicity, only a specific impulse of 450 sec and a constant stage inert mass fraction of 10% of the propellant mass is considered in this paper. For nuclear-electric powerplants, the total mass grows with the power level of the entire system. An expression fitting a wide variety of powerplant designs[4–7] is

$$M_w = 350P^n \tag{1}$$

in which M_W is the mass of the powerplant in kilograms, P is the power level in kilowatts, and the exponent n is equal to 0.6 for the "conservative," 0.5 for the "nominal," and 0.4 for the "advanced" technology estimates. To simplify the comparisons presented in this paper, $n = 0.5$ will generally be used. It is assumed that both the hybrid and the ballistic system utilize a chemical cryogenic Earth escape stage. In the hybrid cases, this stage is used to obtain optimum hyperbolic excess velocity before operation of the electrical stage. For the ballistic cases, capture is obtained by means of a high-thrust rocket with a specific impulse of 300 sec. In the hybrid case and for the capture orbits considered in this paper (circular orbits around the most massive planets), the electrical capture mode yields more payload.

The general capabilities of ballistic and hybrid systems are shown in Fig. 1. The specific mission considered is a Jupiter orbiter with a circular capture orbit at an altitude of 20 planetary radii. Both systems are of continuously variable size. The nuclear electric powerplant follows the scaling law of Eq. (1). The initial mass in Earth orbit (M_0) is used merely to typify vehicle size and by no means implies a necessity to depart from Earth orbit. The straight thin lines represent the capability of the ballistic system for three constant trip times whereas the broken lines represent the capabilities of the hybrid system for the same three trip times. A trip time of 1000 days approximately corresponds to the minimum energy requirement and, therefore, yields maximum payload for the ballistic system in the simple two-impulse mode. For a trip time of roughly 600 days, the ballistic payload vanishes entirely.

At any specific trip time, the ballistic and the hybrid capability lines intersect at the point where both systems can carry the same payload for the same size of launch vehicle in the same trip time. These points of equal performance

Presented as Paper 69-249 at the AIAA 7th Electric Propulsion Conference, Williamsburg, Va., March 3–5, 1969; submitted March 10, 1969; revision received August 4, 1969.

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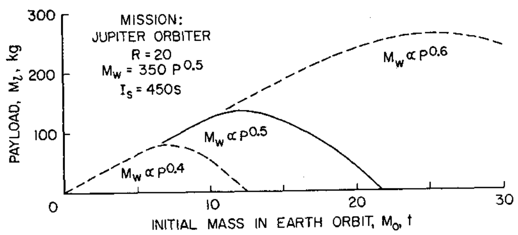

Fig. 2 Boundaries of equal performance for one mission.

establish the bell-shaped heavy line ("equal performance boundary") below which the ballistic systems have a superior payload capability and above which the opposite is true. For the hybrid system, maximum payload is obtained at infinite trip time at which the payload M_l approaches the initial mass M_0. The shaded wedge along the ordinate represents the area within which the hybrid payload would exceed M_0, a physical impossibility for the electrical stages presently under consideration.

In Fig. 2, Eq. (1) has been used with all three different exponents to generate "advanced" and "conservative" boundaries, which are shown as broken curves surrounding the previously derived "nominal" curve. These performance boundaries show that for every set of assumptions there is a maximum size of payload above which hybrid systems always perform better than ballistic systems. The same boundaries indicate that there is also a maximum launch vehicle size above which the hybrid combination always outperforms the ballistic system. Additional conclusions can only be obtained by considering costs as well as other aspects not directly related to performance.

Because of the parametric nature of this paper, no attempt is made to obtain detailed economic comparisons. Note, however, that the possibility of trading off the cost of an electric upper stage against a reduction in launch vehicle size cannot at all be evaluated without making some assumptions about cost relationships. Situations will be found under which the recurrent cost of a launch vehicle can be expressed as a function $b(M)$ of its capability to carry payload. Let us also assume that, under these circumstances, the cost of an electric upper stage can be expressed as a function $e(P)$ of its installed power. A mission may be found for which the cost of the complete ballistic launch system is equal to the cost of the combination of high-thrust and electric stages required to carry the same net payload. It is convenient to relate the cost of the ballistic $[b(M_B)]$ and the hybrid $[b(M_H)]$ vehicles, and the cost of the power level $[e(P)]$ associated with the latter as

$$b(M_B) = b(M_H) + e(P) \tag{2}$$

The cost must include that of the boosters, of the Earth-escape stages and, in the case of the ballistic system, of the

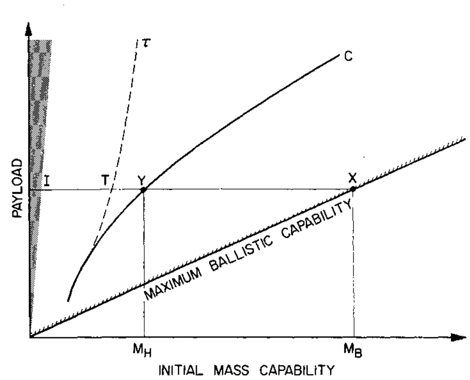

Fig. 3 Hypothetical cost equivalence boundary for one mission.

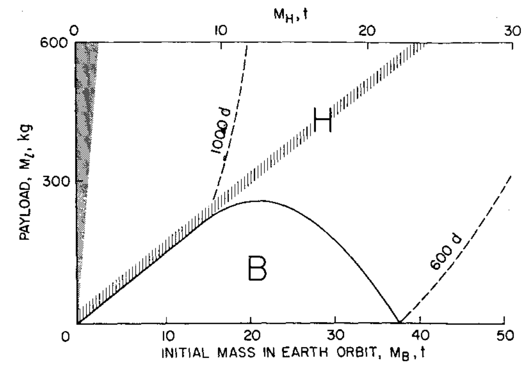

Fig. 4 Performance comparison on an equal cost basis.

capture stage. For simplicity, the same cost coefficient (b) is used in both the hybrid and the ballistic cases. This implies that the cost of the ballistic capture stage is negligibly small and that the payload carries its own source of electric power in both cases; i.e., in the hybrid case, the payload makes no use of the excess power capability created by the electric stage. It is beyond the scope of this short Note to identify those cases in which this simplification will be found inacceptable, and will require special treatment.

Systems optimization studies indicate that the power level for maximized performance is determined primarily by the mass carrying capability of the launch vehicle[5,8] and the specific mass of the powerplant. Since the optimum power level is roughly proportional to M_H, it is possible to rewrite Eq. (2) in terms of only Earth orbital payload capability Eq. (3) whenever the powerplant specific mass is known or can be expressed by a scaling law;

$$b(M_B) = f(M_H) \tag{3}$$

Equation (3), hence, applies whenever the cost of a ballistic launch vehicle (left side) is the same as the cost of a hybrid launch system (right side) of equal mission capability. This is visualized in Fig. 3 where the payload capability of a ballistic vehicle typified by an initial mass capability of M_B is indicated at point X. It is assumed that a hybrid system of initial mass capability M_H was found which matches the payload capabilities of the ballistic system for the same total cost. This is shown as point Y on the diagram. Calling the ratio of the two initial mass capabilities (M_B/M_H) a "tradeoff ratio" and assuming that it can be determined for a range of vehicle sizes, one would then obtain an equal cost boundary shown in hypothetical form by the heavy curve labeled C. Along the broken line labeled τ, the flight time of the hybrid system equals the minimum energy flight time of the ballistic system which is constant. Along the constant payload line described by points I and X, different flight time and cost situations exist. At point I, the smallest feasible hybrid system requires infinite trip time. At point T, a hybrid system which costs less than the ballistic system is capable of carrying the same payload in a flight time equal to the minimum energy flight time of the ballistic system. At point Y, the hybrid flight time is shorter but the cost is identical under the aforementioned assumptions.

Having gained some understanding of some possible effects of launch vehicle tradeoffs and their relationship of the flight time properties of the hybrid mode of transportation, we may now choose a set of cost functions to investigate the possibility of defining an area of superior economic performance of nuclear-electric space propulsion. Obviously, the validity of the results of such a determination will strongly depend on the quality of the cost assumptions fed into the analysis. Since the purpose of this Note is to demonstrate the fundamentals of a methodology rather than to generate conclusions, a completely hypothetical set of cost functions has been adopted. It has been assumed that the cost of placing each metric ton into Earth orbit is independent of launch vehicle size and equals the cost of every 4 kw installed into an electric upper stage, also independent of size. It is important to realize that neither payload cost nor the cost of an electric stage can be expected to be independent of size; the results obtained by means of this simplification can therefore not be of general validity. However, this simple-minded assumption will be found of considerable help in illustrating the method. A circular orbit around Jupiter with an altitude of 20 planetary radii is to be established. Under these conditions, the diagram shown in Fig. 4 may be constructed. As in the previous figures, the payload is plotted along the ordinate whereas the initial mass capability of the ballistic system is plotted along the bottom abscissa. In order to obtain the performance boundary for systems of equal cost, rather than systems of equal initial mass, the horizontal scale at the top of the diagram, which is determined by the tradeoff ratio, has been used for the performance of the hybrid vehicles. Under these conditions, along any vertical line ballistic and hybrid systems exhibit the same recurrent cost. For the hypothetical cost assumptions used in this Note, a hybrid system which equals the cost of a ballistic system must be based on a launch vehicle whose initial mass capability is approximately 48% of the initial mass capability of the ballistic counterpart, i.e., the tradeoff ratio is 0.48. Comparing Fig. 4 to Fig. 1, it becomes evident that the equal performance boundary has been modified by the inclusion of the equal cost groundrule. Specifically, the area of ballistic superior performance has been enlarged in such a way as to roughly double the magnitude of the payload maximum. The shaded line indicates the physical limit of capability for the ballistic system. Below this line, the ballistic system is capable of performing the mission but, outside the area B, requires more trip time than the hybrid system. To further aid in the comparison, the hybrid performance lines for the longest (1000 days) and shortest (600 days) ballistic trip times are also shown in the diagram as broken lines inside H, the area of superior hybrid performance.

Concluding Remarks

A method has been developed which allows one to compare the performances of space propulsion systems of widely differing characteristics. Biased comparisons can be avoided by systematically arranging and selecting the groundrules of the comparison before they are fed into the analysis. Meaningful results can be obtained in spite of the large range of uncertainties attached to some of the performance parameters by adopting the use of scaling laws which reflect major trends or other relationships of interest rather than scattered performance estimates. This Note has intentionally been limited to a description of the method. Quantitative conclusions may be obtained by exercising the use of realistic performance and cost inputs whenever available.

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Solubilities of Gases in Simple and Complex Propellants

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Nomenclature

A	= constant defined by $\Delta H^0/4.5756$, see ΔG^0
B	= constant defined by $\Delta S^0/4.5756$, see ΔG^0
ϵ/k	= force constant in °K for gases obtained from an equation such as Lennard-Jones equation for potential energy vs intermolecular distance, and from the second virial coefficient
ΔG^0	= change in standard Gibbs energy, $\Delta G^0 = \Delta H^0 - T\Delta S^0$, cal/mole
ΔH^0	= change in standard enthalpy in calories per mole for dissolution process
i	= solute, a sparingly dissolved gas in this Note
j	= solvent, a propellant in this Note
K_i	= equilibrium constant; mole fraction of solute i divided by its partial pressure P_i in atmospheres over solution
K_i(ppm)	= concentration, ppm, of i in solution divided by its partial pressure P_i, atm, over solution
n	= number of moles
P	= pressure, atm
ppm	= parts per million of concentration in weight
R	= ideal gas constant; 0.0820537 liter-atm/°K-mole in $PV = RT$, and 1.987165 cal/°K-mole in $\Delta G^0 = -RT \ln K_i$
ΔS^0	= change in standard entropy in calories/mole/°K
T	= temperature, °K
V	= volume, liter
X_i	= mole fraction of i; for simple propellants in this paper $X_i + X_j = 1$
Z	= compressibility factor, dimensionless

Introduction

LIQUID propellants are pressurized with gases for two main reasons: 1) to provide a protective blanket for the propellants, and 2) to eject the propellants from their containers in the space vehicles. This process eliminates the use of heavy pumps and increases the payload. The gases used for pressurization dissolve to various extents[1–3] and cause pressure decay in the containers. The pumps and the venturis desorb the gases in solution in the form of small bubbles and cause undesirable hydrodynamic effects. A knowledge of the solubilities of gases in liquid propellants is therefore very useful.

Experimental Procedure

The apparatus for solubility measurements, shown in Fig. 1, was all Pyrex glass construction with short capillary connecting tubes joined by fusion. Two capillary stopcocks A and B were sparingly lubricated with silicone grease. Three volumes C, D, and E, calibrated to ±0.0003 mliter, were interconnected with short capillary necks, each with a calibration mark. These volumes were immersed in a

Received May 23, 1969.

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